

## **Nonlinear Internal Tide Generation at the Luzon Strait: Integrating Laboratory Data with Numerics and Observation**

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### **LONG-TERM GOALS**

The long-term goal of this project is to develop a reliable predictive capability for the generation of nonlinear internal waves that originate at the Luzon Strait. This is to be achieved using a combination of laboratory experiments and theoretical analysis, and through the pursuit and development of interactions with numerical models and field data.

### **OBJECTIVES**

The objective of the research is to gain fundamental insight into the internal tide generation processes that occur at the Luzon Strait. The outstanding issue is that both the topography and barotropic tides at this location are complex, and the origin of pertinent features such as the large-amplitude, westward-propagating solitary waves is therefore still undetermined.

### **APPROACH**

The experimental approach is to utilize a custom-designed and state-of-the-art wave tank facility to study internal wave tide generation by different topographic features in dynamical regimes that are relevant to Luzon. This is complemented by analytical studies that utilize a Green-function approach to predict the internal tides generated by nominally two-dimensional topographic features. Where possible, there is interaction with the results of field studies and numerical simulations.

### **WORK COMPLETED**

Several aspects of this work have been completed in the last year. A major step forward has been the development of the Green-function-based theoretical approach to internal tide generation by arbitrarily complex two-dimensional topography in nonlinear stratifications. For the first time, this enables reasonable analytical predictions of the internal tide generated by the Luzon Ridge system. This study highlights the ability of double-ridge systems to trap internal wave energy between the ridges, and the model is also capable of accounting for dissipation. The accuracy of the model was confirmed by direct comparison with laboratory experiments. Finally, dynamical systems tools have been developed

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as an efficient means of locating internal wave attractors in a double ridge system, and a survey of the Luzon Ridge system revealed the possibility of attractors near Taiwan.

From this work there have been three submitted manuscripts (Echeverri and Peacock 2009, Echeverri et al. 2009 and Tang and Peacock 2009). Oral presentations have been given at University of Alberta, Caltech, Scripps and the ONR Review Meeting in Chicago. In addition, a proposal was accepted to run an Internal Waves Workshop in Banff in April 2010, which several of the participants of the ONR program will attend (Alford, Sarkar, Farmer, Helfrich, Klymak, MacKinnon). Finally, there is continued interaction with the Discovery Channel on the production of a documentary on Internal Waves that will feature the Aug 2008 NLIWI research cruise.

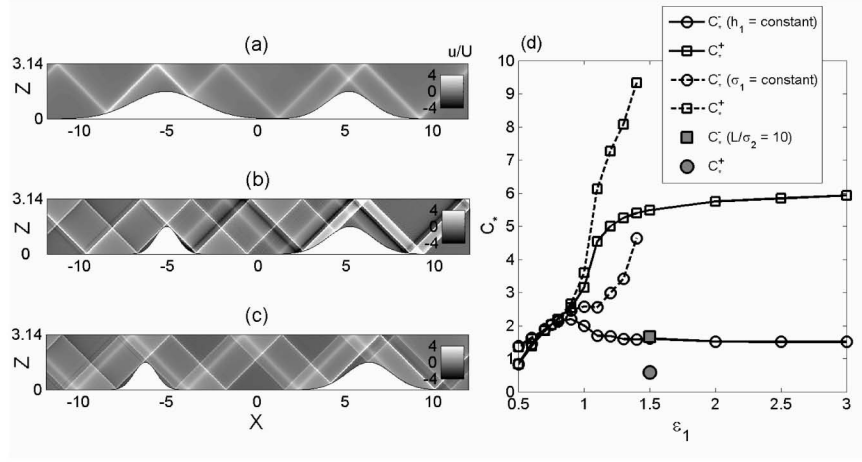
## RESULTS

### 1. Internal tide generation by complex topography

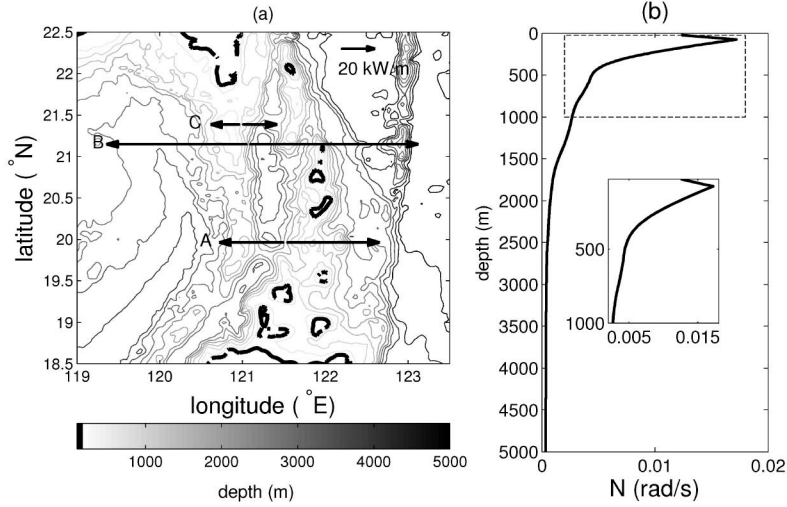
We have made two major advancements to the state-of-the-art in analytical modeling of internal tides. By virtue of the modified Green function approach it is now possible to predict internal tide generation for two-dimensional topography that is arbitrarily complex, and in the presence of a realistic stratification via the WKB approximation.

Fundamentally, we find that for a double ridge system such as Luzon the internal tide generation process can be highly sensitive to the stratification and the geometric configuration, due to interaction between the two ridges. Figure 1 presents results for the horizontal velocity field generated by a system of two Gaussian ridges. In Figures 1(a) and 1(b) the criticality of the left ridge is varied and the wave field is seen to vary dramatically. In Figure 1(c), the left ridge is moved slightly compared to Figure 1(b) and even this small alteration is seen to have a significant effect on the global internal tide. Figure 1(d) presents the dimensionless energy flux to the left ( $C^-$ ) and right ( $C^+$ ) as a function of the criticality of the left ridge. There is a strong left-right asymmetry.

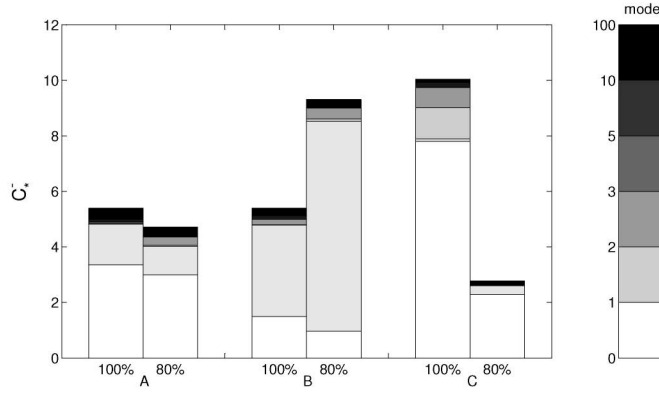
The method was applied to cross sections of the Luzon Ridge, labeled A, B and C in Figure 2(a) for the stratification in Figure 2(b) (data supplied by David Farmer). Figure 3 summarizes the effect on the westward internal tide of decreasing the height of the West Ridge by 20% for all three cross sections. The bar height represents the dimensionless conversion rate, and each bar is decomposed into contributions from the different vertical modes. The energy flux at cross-sections A and C is dominated by mode 1, but at cross section B it is mode 2 that dominates, which is surprising, because on inspection cross section B does not differ so dramatically in form from cross section A. Reducing the height of the west ridge reduces the energy flux for both A and C, dramatically so for C, but substantially increases the energy flux for cross section B. Consistent with the numerical results of Chao et al. (2007), we therefore conclude that at the southern end of the Luzon Strait the West Ridge plays a relatively minor role in internal tide generation due to its small size, whereas further north its role is much more substantial. Their conclusion that at north latitudes the West Ridge augments the westward internal tide seems to be too sweeping a generalization, however, as we find the opposite to be true for cross section C. Our conclusion is that the nature of the radiated internal tide depends very sensitively on the bathymetry and the stratification, and needs to be assessed on a case-by-case basis.



**Figure 1: (a)-(c) Internal tides radiated by different configurations of a double ridge system. A small shift in the position of the left ridge (i.e. (b) compared to (c)) creates a substantial difference in the radiated wave field. (d) The dimensionless energy-flux as a function of the criticality of the left ridge. There is a distinct asymmetry between the left- ( $C^-$ ) and right-going ( $C^+$ ) energy flux.**



**Figure 2: (a) Bathymetry and (b) stratification of the Luzon Strait. The vectors in (a) are proportional to the two-dimensional energy flux evaluated at 3 representative cross sections labeled A, B and C.**



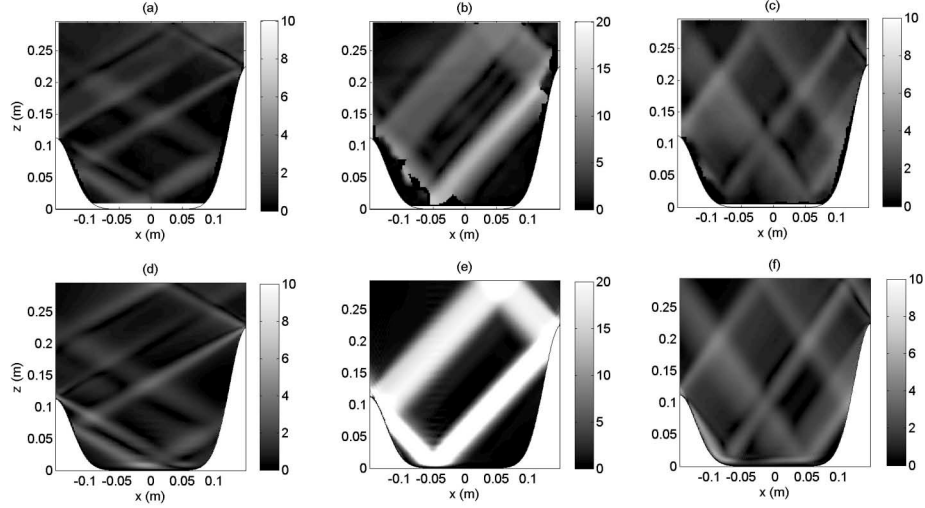
**Figure 3: Dimensionless westward energy flux at locations A, B and C for the case in which the western ridge is at 100% and at 80% of its true height. The fraction of the conversion rate in modes  $p = 1; 2; 3; 5; 10$  and  $p > 10$  is indicated within each bar.**

## 2. Internal tide attractors in double-ridge systems

Through a combination of theory and experiment, we have demonstrated the existence of internal tide attractors in double-ridge systems. The attractor arises because of geometric focusing and trapping of the internal wave rays by the topography. In order to predict their occurrence, we developed a 1D checkerboard map that revealed the existence and stability of attracting closed orbits as a function of the governing system parameters. In the presence of an attractor, the Green function technique for steady state, inviscid internal tide generation fails because the continual focusing of energy onto the attractor ensures no time-periodic solution exists. We found that incorporating dissipation into the method regularizes the solutions in a manner that is independent of the magnitude and form of the damping. Furthermore, the weakly viscous solutions compare well with laboratory experiments, as demonstrated by the results in Figure 4.

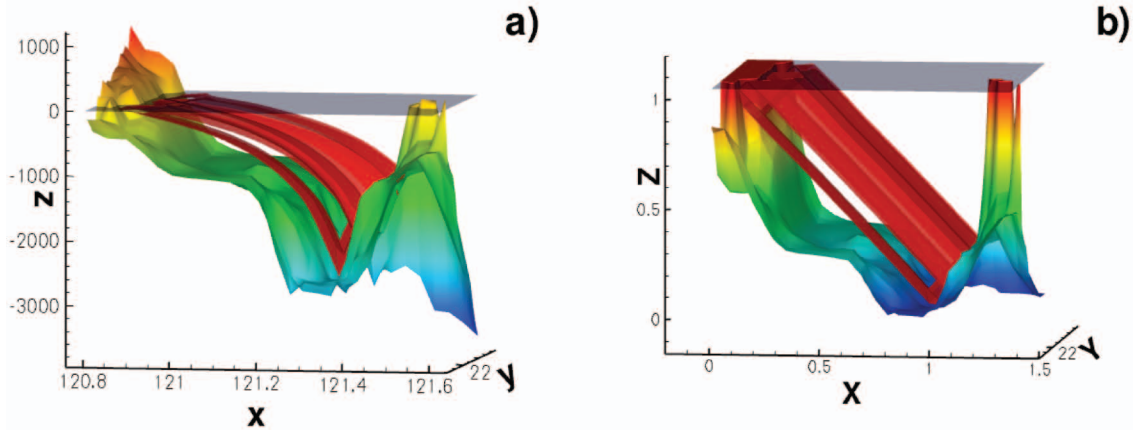
## 3. Lagrangian coherent structures and internal tide attractors.

We have developed an efficient mathematical and numerical tool to find internal wave attractors between two-dimensional parallel submarine ridges. These tools are significantly built on concepts from the study of Lagrangian Coherent Structures, where the geometric pattern of mixing is extracted based on the Lagrangian ray trajectories. WKB stretching is invoked to deal with nonuniform stratification in the ocean. In the stretched coordinate, wave inclinations are at 45 degrees and attraction occurs only upon each reflection at the ocean floor, so the problem can be reduced to stability analyses of fixed points on Poincare sections. Using this approach, we find the parameter dependence of basins of attraction for various types of wave attractors in idealized double ridge configurations.



**Figure 4:** *Experimental and theoretical snapshots of the vertical velocity magnitude between a pair of Gaussian ridges. Panels (a)-(c) present experimental results for beam angles of 29.4, 47.1 and 54.9 degrees, respectively, and panels (d)-(f) show the corresponding viscous theoretical results.*

We successfully applied this approach to the study of wave attractors for the Luzon Ridge. Based on our understandings of the geometric dependence of attractors, the distance between the two ridges in mid-section of the Luzon ridge system falls between so-called (1,1)- and (2,1)-attractors and the West Ridge is not significantly supercritical, undermining the possibilities of wave attractors in these regions. However, a significant northern portion of the Luzon Ridge can support (1,1)-attractors, as shown in Figure 5. By studying several density profiles, we have also found the dependence of wave attractors on stratifications. In general, summer stratification is more favorable in supporting wave attractors than winter stratification, due to the change in mean stratification which effectively results in the changing ridge separation.



**FIG. 6:** *Internal wave attractor near Luzon using the GDEM density profile. Marked in red are the attracting surfaces. (a) Physical coordinates. (b) WKB stretched coordinate.*

## IMPACT/APPLICATIONS

There are several significant impacts of this work. First, the newly developed analytical method, which has been validated by laboratory experiments, provides an efficient way of reasonably calculating the internal tide radiated by the Luzon ridge. These calculations can be run in a few minutes on a laptop computer using MATLAB, providing sea-going oceanographers with a practical tool to use for preliminary studies of internal tide generation, before a computationally-expensive three-dimensional simulation is run. The studies we have performed show that there is likely a very sensitive interaction between the East and West Ridges at Luzon, implying that good knowledge of both the stratification and the bathymetry is essential for high quality predictions. Finally, the work on internal tide attractors suggests that we can expect highly active internal tides in the region between the ridges at Luzon.

## RELATED PROJECTS

The work for this project connects to the work we have started for the ONR IWISE program, in which we are seeking to investigate three-dimensional internal tide generation. In addition it connects to ongoing work we have on experimental studies of internal wave generation using a novel wave generator and internal tide scattering.

## REFERENCES

1. Chao, S.Y., Ko, D.S., Lien, R.C. and Shaw, P.T., “Assessing the west ridge of Luzon Strait as an internal wave mediator,” *Journal of Oceanography* 63, 897-911 (2007).

## PUBLICATIONS

1. Echeverri, P. and Peacock, T., “Internal tide generation by complicated topography,” submitted to *Journal of Fluid Mechanics* (17 pages).

2. Echeverri, P., Yokossi, T., Balmforth, N.J. and Peacock, T., “Internal tide attractors in double ridge systems,” submitted to *Journal of Fluid Mechanics* (18 pages).

3. Tang, W.E. and Peacock, T., “Lagrangian coherent structures and internal tide attractors,” submitted to *CHAOS* (10 pages).